Probability exercises - Problem Set 6 - Continuous Random Variables and Random Vectors

Manuel Lleonart Anguix

For this Problem Set I added the notation **!**. It means that an exercise is more difficult than the others in the Problem Set. **!!**, as you might imagine, is translated to *only for try-harders*.

Exercise 1. *Let $\boldsymbol{X} = \{X_1, x_2, ..., X_n\}$ be a random sample i.i.d. Find the maximum likelihood estimator of the following parameters.*

1. *μ of a $N(\mu, 1)$.*
2. ***!** σ^2 of a $N(0, \sigma^2)$.*
3. ***!** μ and σ^2 of a $N(0, \sigma^2)$.*
4. *λ for a $Po(\lambda)$.*
5. *p for a **geometric distribution** of parameter p, $Ge(p)$.*
6. *θ for a $Un(0, \theta)$.*
7. *θ for a $Un(\theta, 1)$.*
8. *θ for a $Un(\theta)$ (here θ is a vector of dimension two).*
9. *p for a Bernouilli distribution.*
10. *p for a Binomial distribution.*
11. *λ for an Exponential distribution $Ex\left(\frac{1}{\lambda}\right)$.*

Exercise 2. *Show by definition or by the factorization criteria that the previous are sufficient estimators.*

Exercise 3. ***!*** *Let $\boldsymbol{X} = X_1 - X_2$. Where X are i.i.d. with distribution function $X \sim Ge(\theta)$. Show that the statistic $T(X) = X_1 - X_2$ is not a sufficient statistic by definition.*